Astronomical Approximations. I. Apparent Semi-diameter of the Sun, and Nebular Origin of the Terrestrial Day. By Pliny Earle Chase, LL.D., Professor of Philosophy in Haverford College.

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The various mathematical deductions which I have drawn from the nebular hypothesis, as modified by Herschel's theory of "subsidence," furnish many independent tests for judging of the probable accuracy of delicate and difficult astronomical observations. The consistency of the tests may be shown by examples, one of which is given in the present note.

The hypothesis that the solar system has been shaped by undulations, moving with the velocity of light, leads to the following equation:

$$\left(\frac{1 \text{ year}}{1 \text{ day}}\right)^3 = \frac{\pi^2}{2} \times \left(\frac{\text{Earth's distance}}{\text{Sun's semi-diameter}}\right)^3$$

From this equation we find, for Earth's mean distance from Sun, 214.54 solar semi-diameters; and for Sun's apparent diameter, 32′ 2″.85. The accordance of this result with observations is as follows:

	Distance.	Diameter.
Newcomb,	214.86	32′ 0″
Chase,	214.54	32 2.85
Fuhg,	214.52	32 2.99
British Naut. Almanac,	214.45	32 3.64
American " "	214.41	32 4
Lockyer,	214.39	32 4.205

Dr. Fuhg's estimate, which approximates most closely to my own, is deduced from 6827 measurements.* Notwithstanding the vast labor which is represented by those measurements and their subsequent discussion, I cannot but believe that my own result is still more accurate. For it involves no careful micrometry, no allowance for irradiation, and no other elements of possible uncertainty than small fractions of a second, in the estimated lengths of the sidereal year and of the mean solar day.

This result may, perhaps, be rightly regarded as an *experimentum crucis*. Therefore, to avoid the trouble of referring to the papers in which I have established the data for my formula, I will repeat the fundamental considerations on which it rests.

Any body, revolving in a circular orbit, under the influence of a central force g, which varies inversely as the square of the distance, would acquire the velocity of revolution $(\sqrt{gr})$, in the time of describing an arc equivalent to radius. It would acquire a parabolic velocity $\sqrt{2gr}$, in $\frac{1}{\pi\sqrt{2}}$ of a revolution, and it would acquire $\frac{\pi}{\sqrt{2}}$ times the parabolic velocity in a half revolution, provided all the increments of the central

* *Astron. Nachrichten*, 2010, cited in *Am. Journ. Sci.* for Aug. 1875, p. 57.

force were retained. The parabolic velocity is the limit between the tendencies to incipient aggregation and to complete dissociation, in the subsidence of dense nebulous particles.

In a body which is both revolving about the centre of a system and rotating on its own axis, every particle is subject, during each half-rotation, to cyclical variations in the systematic stress, which are antagonized by the constant central force of the body itself.

In a condensing or expanding nebula, the time of rotation varies as the square of radius. At any given instant, the squares of the velocities, or the living forces of different rotating particles, are proportioned to the squares of their radii, or to the times of nebular rotation and synchronous revolution when condensed to their respective positions.

If the velocity of synchronous rotation and revolution is determined by the aggregate resistances of the central force of the nucleus, to cyclical variations of stress which may be represented by a wave-velocity, while the velocity of rotation of a detached planet is determined by a force which may be represented by an incipiently aggregating or parabolic velocity, the representative parabolic velocity of the nucleus may be found by taking $\frac{\sqrt{2}}{\pi} gt$; g representing the force of gravity at any past, present, or future equatorial limit of the nebula, or Laplace's limit of possible rotating atmosphere, and t being $\frac{1}{2}$ the corresponding time of synchronous rotation and revolution. I have already shown that gt, in the solar system, is *the velocity of light.*

Let α represent the time of synchronous rotation and revolution when Sun's surface should contract so that Laplace's limit would correspond with its present equatorial radius; β, a mean solar day; γ, Earth's mean distance $\div$ Sun's radius; δ, a sidereal year. Then $\alpha = \delta \div \gamma^{\frac{3}{2}}$; tang. Sun's apparent semi-diameter $= \frac{1}{\gamma}$.

According to Leverrier and Hansen, Earth's present aphelion, or limit of incipient condensation, is at 1.016771 rad. vec. This corresponds, within $\frac{1}{100}$ of one per cent., to Stockwell's determination of the centre of the belt of greatest condensation.* The relations between the primitive solar and terrestrial centres are thus simplified, so that the foregoing considerations lead us at once to the following equations:

$$\left(\frac{\delta}{\beta}\right)^3 = \frac{\pi^2}{2}\gamma^3$$

$$\frac{\delta}{\beta} = \frac{\pi^2}{2}\left(\frac{\beta}{\alpha}\right)^2$$

*The arithmetical mean between Mercury's secular perihelion (.2974307) and Mars's secular aphelion (1.7363251) is 1.016878.